

# Online Research @ Cardiff

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <https://orca.cardiff.ac.uk/id/eprint/112878/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Sanchez-Monedero, Javier ORCID: <https://orcid.org/0000-0001-8649-1709>, Gutiérrez, Pedro A., Fernández-Navarro, F. and Hervás-Martínez, C. 2011. Weighting efficient accuracy and minimum sensitivity for evolving multi-class classifiers. Neural Processing Letters 34 (2) , 101. 10.1007/s11063-011-9186-9 file

Publishers page: <http://dx.doi.org/10.1007/s11063-011-9186-9>  
<<http://dx.doi.org/10.1007/s11063-011-9186-9>>

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies.

See

<http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



# Weighting efficient Accuracy and Minimum Sensitivity for evolving multi-class classifiers

Javier Sánchez-Monedero · Pedro A. Gutiérrez · F. Fernández-Navarro · C. Hervás-Martínez

Received: date / Accepted: date

**Abstract** Recently, a multi-objective Sensitivity-Accuracy based methodology has been proposed for building classifiers for multi-class problems. This technique is especially suitable for imbalanced and multi-class datasets. Moreover, the high computational cost of multi-objective approaches is well known so more efficient alternatives must be explored. This paper presents an efficient alternative to the Pareto based solution when considering both Minimum Sensitivity and Accuracy in multi-class classifiers. Alternatives are implemented by extending the Evolutionary Extreme Learning Machine algorithm for training artificial neural networks. Experiments were performed to select the best option after considering alternative proposals and related methods. Based on the experiments, this methodology is competitive in Accuracy, Minimum Sensitivity and efficiency.

**Keywords** Artificial Neural Networks · Extreme learning machine · Evolutionary ELM · Multi-class · imbalanced datasets · Accuracy · Sensitivity · Differential Evolution

## 1 Introduction

Global performance measures such as the Correct Classification Rate (CCR) or Accuracy are not enough to evaluate classifiers [20]. Even in two class problems, Accuracy reflects a one-dimensional ordering where two different types of errors can be found. Alternative measures are especially needed in imbalanced datasets – problems where the number of patterns in each class is significantly different –.

Martínez et. al. [18] address the problem of the one dimensional consideration in multi-class problems. In this work, two measures are considered to evaluate a classifier: traditionally used accuracy ( $C$ ) and the minimum sensitivities of all

---

Javier Sánchez-Monedero  
Department of Computer Science and Numerical Analysis, University of Córdoba, Rabanales Campus, Albert Einstein building 3rd floor, 14071, Córdoba, Spain  
Tel.: +34-957-218349  
Fax: +34-957-218360  
E-mail: jsanchezm@uco.es

classes ( $MS$ ); that is, the premise assumed is that a good classifier should combine a high correct classification rate level in the generalization set with an acceptable accuracy level for each class. For further details about these measures please check [18] and [13].

We consider a classification problem with  $Q$  classes and  $N$  training patterns with  $g$  as a classifier obtaining a  $Q \times Q$  contingency or confusion matrix  $M(g) = \{n_{ij}; \sum_{i,j=1}^Q n_{ij} = N\}$  where  $n_{ij}$  represents the number of times the patterns are predicted by classifier  $g$  to be in class  $j$  when they really belong to class  $i$ .

Two scalar measures are defined considering different points of view of the elements in the confusion matrix. Let us denote the number of patterns associated with class  $i$  by  $n_i = \sum_{j=1}^Q n_{ij}$ ,  $i = 1, \dots, Q$ . Let  $S_i = n_{ii}/n_i$  be the number of patterns correctly predicted to be in class  $i$  with respect to the total number of patterns in  $i$  (sensitivity  $S_i$  for class  $i$ ). Therefore, the sensitivity for class  $i$  estimates the probability of correctly predicting a class  $i$  example. From the above quantities, Sensitivity  $MS$  of the classifier is defined as the minimum value of the sensitivities for each class,  $MS = \min \{S_i; i = 1, \dots, Q\}$ . The Correct Classification Rate or Accuracy is defined as  $C = (1/N) \sum_{j=1}^Q n_{jj}$ , which is the rate of all correct predictions.

These measures have recently been proposed to address evolutionary training for multilayer perceptron (MLP) neural networks [13]. In [13]  $C$  and  $MS$  are presented as objectives which can be competitive. This fact justifies the use of an evolutionary multi-objective algorithm to train artificial neural networks (ANNs). The results are presented by showing the performance, in terms of  $C$  and  $MS$  measures of the individuals belonging to the two extremes of the Pareto front for  $C$  and  $MS$ .

In general, a weighted combination of objective functions does not necessary perform better (in accuracy) than a Pareto approach, performing worse in some cases [22]. However, it is well known that they are computationally costly [9] and weighted combination can be a more efficient alternative, specially when only two objectives are considered. In addition, these algorithms' output is a set of possible solutions to the problem; thus, an expert or a decision-making system is needed to select the proper solution belonging to the Pareto front. To overcome these issues, this paper attempts to improve the minimum sensitivity of binary and multi-class classifiers by using a weighted linear combination as an alternative to the Pareto based solution. The Evolutionary Extreme Learning Machine (E-ELM) algorithm [27] is extended with different fitness functions which are automatically adjusted to each dataset by the proposed algorithm.

This article is a significant extension and improvement of a previous work [23]. The fitness function design has been extended from a discrete function to a continuous one with improved results. In addition the fitness function  $\lambda$  parameter is automatically selected by the revised algorithm so that no extra parameters are needed.

The rest of the paper is organized as follows. Section 2 briefly presents ELM, Evolutionary ELM, and proposes different fitness functions and the Evolutionary ELM considering  $C$  and  $MS$  (E-ELM-CS) algorithm. Section 3 shows experiments comparing several proposed fitness functions and different related state of the art methods. Finally, some conclusions are drawn.

## 2 Proposed Method

### 2.1 Differential Evolution and Extreme Learning Machine

The Extreme Learning Machine (ELM) algorithm has been proposed by Huang [15, 14]. It has been successfully applied to problems such as QoS Violation detection in multimedia transmission [8] or microarray gene expression cancer diagnosis [26]. This section briefly presents ELM algorithm and the Evolutionary ELM.

Let us consider the training set given by  $N$  samples  $D = \{(\mathbf{x}_j, \mathbf{y}_j) : \mathbf{x}_j \in R^K, \mathbf{y}_j \in R^Q, j = 1, 2, \dots, N\}$ , where  $\mathbf{x}_j$  is a  $K \times 1$  input vector and  $\mathbf{y}_j$  is a  $Q \times 1$  target vector.

Let us consider the MLP with  $M$  nodes in the hidden layer given by  $f(\mathbf{x}, \boldsymbol{\theta}) = (f_1(\mathbf{x}, \boldsymbol{\theta}_1), f_2(\mathbf{x}, \boldsymbol{\theta}_2), \dots, f_Q(\mathbf{x}, \boldsymbol{\theta}_Q))$ :

$$f_l(\mathbf{x}, \boldsymbol{\theta}_l) = \beta_0^l + \sum_{j=1}^M \beta_j^l \sigma_j(\mathbf{x}, \mathbf{w}_j), l = 1, 2, \dots, Q, \quad (1)$$

where  $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_Q)^T$  is the transpose matrix containing all the neural net weights,  $\boldsymbol{\theta}_l = (\beta_0^l, \mathbf{w}_1, \dots, \mathbf{w}_M)$  is the vector of weights of the  $l$  output node,  $\beta^l = \beta_0^l, \beta_1^l, \dots, \beta_M^l$  is the vector of weights of the connections between the hidden layer and the  $l$ th output node,  $\mathbf{w}_j = (w_{1j}, \dots, w_{Kj})$  is the vector of weights of the connections between the input layer and the  $j$ th hidden node,  $Q$  is the number of classes in the problem,  $M$  is the number of sigmoidal units in the hidden layer and  $\sigma_j(\mathbf{x}, \mathbf{w}_j)$  the sigmoidal function:

$$\sigma_j(\mathbf{x}, \mathbf{w}_j) = \frac{1}{1 + \exp\left(-\left(w_{0j} + \sum_{i=1}^K w_{ij}x_i\right)\right)}, \quad (2)$$

where  $w_{0j}$  is the bias of the  $j$ th hidden node. Suppose that a MLP is being trained with  $M$ -nodes in the hidden layer to learn the  $N$  samples of set  $D$ . The linear system  $f(\mathbf{x}_j) = \mathbf{y}_j, j = 1, 2, \dots, N$ , can be written in a more compact format as  $\mathbf{H}\boldsymbol{\beta} = \mathbf{Y}$ , where  $\mathbf{H}$  is the hidden layer output matrix of the network:

$$\mathbf{H}(\mathbf{x}_1, \dots, \mathbf{x}_N, \mathbf{w}_1, \dots, \mathbf{w}_M) = \begin{bmatrix} \sigma(\mathbf{w}_1 \cdot \mathbf{x}_1) & \cdots & \sigma(\mathbf{w}_M \cdot \mathbf{x}_1) \\ \vdots & \ddots & \vdots \\ \sigma(\mathbf{w}_1 \cdot \mathbf{x}_N) & \cdots & \sigma(\mathbf{w}_M \cdot \mathbf{x}_N) \end{bmatrix}_{N \times M},$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_M \end{bmatrix}_{M \times Q} \quad \text{and} \quad \mathbf{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}_{N \times Q}$$

The ELM algorithm randomly selects the  $\mathbf{w}_j = (w_{1j}, \dots, w_{Kj}), j = 1, \dots, M$ , weights and biases for hidden nodes, and analytically determines the output weights  $\beta_0^l, \beta_1^l, \dots, \beta_M^l$  for  $l = 1 \dots Q$  by finding the least square solution to the given linear system. The minimum norm least-square solution (LS) to the linear system is  $\hat{\boldsymbol{\beta}} = \mathbf{H}^\dagger \mathbf{Y}$ , where  $\mathbf{H}^\dagger$  is the Moore-Penrose (MP) generalized inverse of matrix  $\mathbf{H}$ . The minimum norm LS solution is unique and has the smallest norm among all the LS solutions.

The Evolutionary Extreme Learning Machine (E-ELM) [27] improves the original ELM by using a Differential Evolution (DE) algorithm. Differential Evolution was proposed by Storn and Price [24] and it is known as one of the most efficient evolutionary algorithms which many applications such as artificial neural networks

training [17]. The E-ELM uses DE to select the input weights between input and hidden layers and the Moore-Penrose generalized inverse to analytically determine the output weights between hidden and output layers.

## 2.2 Fitness function design

As mentioned at the introduction, our approach tries to build classifiers with simultaneously optimized  $C$  and  $MS$ . Since these objectives are not always cooperative [13, 18], an evolutionary multi-objective approach could be used.

In this work, a linear combination of these objectives is used to obtain the maximization of objectives  $C$  and  $MS$ . This option is a good alternative to proper multi-objective evolutionary algorithms when there are two objectives and when the first Pareto front has a very small number of models. In addition, its computational cost is noticeably lower.

Weighted linear combination proves to be very efficient in practice for certain types of problems, for example in combinatorial multi-objective optimization. Some of the applications of this technique are to schedule evaluation of a resource scheduler or to design multiplierless infinite impulse response (IIR) filters [9].

We assume we do not have a priori information about the proper weighting of  $C$  and  $MS$  for each dataset. Thus, both measures are considered equally important. Thereby, we deal with this problem by adapting the algorithm to each dataset through a nested cross-validation procedure. Our purpose is to design a fitness function able to weight up both  $C$  and  $MS$  objectives in the algorithm.

There is no rule for establishing priorities between  $C$  and  $MS$ . Thus, we include a parameter for weighing the two objectives. The underlying idea is to have an automatically adjustable fitness function which could be optimized for each dataset via this parameter, called  $\lambda$ , ranging between  $[0, 1]$ . In this work, three fitness functions are proposed to try to balance the two objectives.

The first fitness function is based on  $C$  and  $MS$ . This function evaluates the performance of a classifier depending on a weighted Accuracy level and a weighted Sensitivity. It is defined by:

$$F_{\lambda CS} = (1 - \lambda)C + \lambda MS, \quad (3)$$

According to [4, 5], in general terms, the use of continuous function for training neural networks for classification problems makes the convergence of the algorithm more robust. By using the root mean square error (RMSE) or the cross-entropy error [4] the fitness function is turned into a continuous function.

In order to properly calculate the RMSE and the cross-entropy error, the neural network outputs need to be interpreted as probabilities  $p_l$ , thereby, they must satisfy the following constraints [4]:

$$\sum_{l=1}^Q p_l(\mathbf{x}, \boldsymbol{\theta}_l) = 1, \quad (4)$$

$$0 \leq p_l(\mathbf{x}, \boldsymbol{\theta}_l) \leq 1. \quad (5)$$

The first constraint also ensures that the distribution is correctly normalized, so that  $\int p(\mathbf{y}|\mathbf{x})d\mathbf{y} = 1$ . These constraints can be satisfied by choosing a  $p_l$  output

to be related to corresponding network outputs  $f_l$  by a softmax function [6]. Then, the softmax activation function is added to standard ELM model outputs:

$$p^l = p_l(\mathbf{x}, \boldsymbol{\theta}_l) = \frac{\exp(f_l(\mathbf{x}, \boldsymbol{\theta}_l))}{\sum_{i=1}^Q \exp(f_i(\mathbf{x}, \boldsymbol{\theta}_i))}, 1 \leq l \leq Q, \quad (6)$$

where  $f_l$  are the ELM outputs defined in Eq. 1 and  $p_l$  is the posterior probability that a pattern  $\mathbf{x}$  has of belonging to class  $l$ . A pattern will belong to the class with the greatest membership probability, this is:

$$C(\boldsymbol{\theta}_l, \mathbf{x}) = \arg \max_l p_l(\mathbf{x}, \boldsymbol{\theta}_l), 1 \leq l \leq Q. \quad (7)$$

Then, once a probabilistic function from the ELM output is determined, a fitness function based on RMSE is proposed:

$$F_{\lambda R}(\boldsymbol{\theta}) = (1 - \lambda) \frac{1}{1 + \frac{1}{N} \sum_{l=1}^Q (n_l R_l(\boldsymbol{\theta}))} + \lambda \frac{1}{1 + \max \{R_l(\boldsymbol{\theta}), l = 1, \dots, Q\}}, \quad (8)$$

where  $n_l = \sum_{i=1}^Q n_{il}$ ,  $i = 1, \dots, Q$  is the number of patterns associated with class  $l$ .  $R_l(\boldsymbol{\theta})$  is the RMSE per class in the problem, defined by:

$$R_l(\boldsymbol{\theta}) = \sqrt{\frac{\sum_{i=1}^N \sum_{l=1}^Q (y_i^l - p_i^l(\mathbf{x}_i, \boldsymbol{\theta}))^2}{n_l Q}}, \quad (9)$$

where  $N$  is the number of patterns,  $y_i^l$  is the target value for class  $l$  of pattern  $\mathbf{x}_i$  ( $y_i^l$  will be equal to 1, in terms of probability, if the pattern  $\mathbf{x}_i$  belongs to class  $l$  and 0 otherwise),  $p_i^l$  is the probability pattern  $\mathbf{x}_i$  has of belonging to class  $l$ , and  $n_l$  is the number of patterns associated with class  $l$ .

The fitness function defined in Eq. 8 introduces an alternative for considering  $C$  and  $MS$ . The first term represents the global accuracy error for all the classes while the second term represents the isolated error of the worst classified class as defined in Eq. 9. The maximum error is selected because it is the equivalent to consider sensitivity, which is the minimum accuracy for each class.

The third fitness function proposed is based on cross-entropy error in a similar way to  $F_{\lambda R}$  defined in Eq. 8:

$$F_{\lambda E} = (1 - \lambda) \frac{1}{1 + \frac{1}{N} \sum_{l=1}^Q (n_l E_l)} + \lambda \frac{1}{1 + \max \{E_l, l = 1, \dots, Q\}}, \quad (10)$$

where  $E_l$  is the cross-entropy error per class in the problem, defined by:

$$E_l = \frac{-\sum_{i=1}^N \sum_{l=1}^Q (y_i^l \ln(p_l(x_i, \boldsymbol{\theta}_l)))}{n_l}, \quad (11)$$

This error function is also known as the negative log likelihood and, when it is minimized, maximum likelihood estimates ( $p_l(\mathbf{x}_i, \boldsymbol{\theta}_l)$ ) are obtained for the event observed.

---

**Require:** E-ELM-CS (P (Training Patterns), T (Training Tags), F (Fitness Function),  $\lambda$ )

```

1:  $\hat{\lambda} \leftarrow$  Calculate optimal  $\lambda$  for F and (P,T)
2: Create a random initial population  $\theta = [\mathbf{w}_1, \dots, \mathbf{w}_k, b_1, \dots, b_k]$  of size  $N$ 
3: for each individual do
4:    $\hat{\beta} \leftarrow$  ELM_output( $\mathbf{w}, P, T$ ) {Calculate output weights}
5:   fitness  $\leftarrow$  getFitness( $\mathbf{w}, \hat{\beta}, F, \hat{\lambda}, P, T$ ) {Evaluate individual}
6: end for
7: Select best individual of initial population
8: while Stop condition is not met do
9:   Mutate random individuals and apply crossover as described in [27]
10:  for each individual in the new population do
11:     $\hat{\beta} \leftarrow$  ELM_output( $\mathbf{w}, P, T$ ) {Calculate output weights}
12:    fitness  $\leftarrow$  getFitness( $\mathbf{w}, \hat{\beta}, F, \hat{\lambda}, P, T$ ) {Evaluate model}
13:    Select new individuals for replacing individuals in old population
14:  end for
15:  Select the best model in the generation
16: end while
17: return Best ELM model

18: function  $\hat{\beta} = \text{ELM\_output}(\mathbf{w}, P, T)$ 
19: Calculate the hidden layer output matrix  $\mathbf{H}$ 
20: Calculate the output weight  $\hat{\beta} = \mathbf{H}^\dagger \mathbf{Y}$ 
21: return  $\hat{\beta}$ 

22: function  $F_\lambda = \text{getFitness}(\mathbf{w}, \beta, F, \lambda, P, T)$ 
23: if  $F_{\lambda CS}$  then
24:   Build training confusion matrix  $\mathbf{M}$ 
25:   Calculate  $C$  and  $S$  from  $\mathbf{M}$ 
26:   Get classifier fitness with Eq. (3)
27: else if  $F_{\lambda R}$  or  $F_{\lambda E}$  then
28:   Add softmax layer to the ELM model ( $\mathbf{w}, \hat{\beta}$ )
29:   if  $F_{\lambda R}$  then
30:     Get classifier fitness with Eq. (8)
31:   else
32:     Get classifier fitness with Eq. (10)
33:   end if
34: end if
35: return Individual fitness

```

Fig. 1: E-ELM-CS algorithm pseudocode.

### 2.3 The E-ELM-CS Algorithm

Our proposed method is implemented by using the Evolutionary ELM (E-ELM)[27]. E-ELM for classification problems only considers the misclassification rate of the classifier. Further details about the algorithm can be consulted in [27]. The E-ELM has been extended in two ways. First, the three fitness functions designed in Section 2.2 are added, including the addition of the softmax layer to the model. Secondly, a 10-fold cross-validation is applied, using exclusively the training data, which aims to optimally configure the  $\lambda$  parameter of the fitness function. Note that after several experiments with different  $\lambda$  values no generic optimal value for  $\lambda$  is found to maximize both  $C$  and  $MS$ , that is,  $\lambda$  depends on the data set. Therefore, a cross-validation process is mandatory for each different dataset. Then, the algorithm extension is called E-ELM-CS (Evolutionary ELM

Table 1: Datasets used for the experiments

Dataset	Size	#Input	#Classes	Distribution	$p^*$
Two classes					
BreastC-W	699	9	2	(458,241)	0.3448
Card	690	51	2	(307,383)	0.4449
Hepatitis	155	19	2	(32,123)	0.2069
Multi-class					
Balance	625	4	3	(288,49,288)	0.0784
Gene	3175	120	3	(762,765,1648)	0.2400
Iris	150	4	3	(50,50,50)	0.3333
Lymph	148	38	4	(2,81,61,4)	0.0135
Anneal	898	59	5	(8,99,684,67,40)	0.0089
Glass	214	9	6	(70,76,17,13,9,29)	0.0421
Zoo	101	16	7	(41,20,5,13,4,8,10)	0.0396

considering  $C$  and  $MS$ ). The E-ELM-CS algorithm pseudocode is shown in Fig. 1. Mutation, crossover and selection operators work as described in [27].

The cross-validation is performed by testing a range of  $\lambda$  values for the chosen fitness function in E-ELM-CS and a given configuration for the remaining parameters. The training set is stratified into 10 sets so 10 validation configurations can be formed. Each one of the 10 validation tests consists of different combinations of 9 sets for training and a different one for validation. Note that generalization data is never used during this cross-validation procedure so that the final algorithm performance will be measured only with unseen data. For each  $\lambda$  value to validate, the E-ELM-CS algorithm is run three times with the same data partition. Therefore the total number of executions over the training data is 30. The  $\lambda$  considered as optimal is the one shown in the following equation:

$$\hat{\lambda} = \arg \max_{\lambda_i} \frac{\overline{C_{\lambda_i}} + \overline{MS_{\lambda_i}}}{2}, \quad (12)$$

where  $\overline{C_{\lambda_i}}$  is the mean  $C$  and  $\overline{MS_{\lambda_i}}$  is the mean  $MS$  obtained by the algorithm in the different validation folds using  $\lambda_i$  for the fitness function.

The parameters related to the evolutionary algorithm are the same for the whole cross-validation process with the exception of the parameter related to the number of generations, which is reduced to 1/5 of the final number of generations because, experimentally, it is not necessary to go further in the number of generations in order to find the best  $\lambda$ . The  $\lambda$  values to be tested are selected from the range  $[0, 1]$  in intervals of 0.25. Previous experiments confirmed that there were no significant differences if the cross-validation was performed with more values. So, considering a few  $\lambda$  values and reducing the number of generations in the algorithm, the cross-validation process time is drastically reduced.

### 3 Experiments

The purpose of the experiments is to evaluate which fitness function is more suitable for E-ELM-CS with the purpose of simultaneous optimization of  $C$  and  $MS$ .



Computational cost, in terms of training time  $T$ , is also considered. Results are compared with related state of the art methods.

There were ten UCI repository datasets with different features under study [3] (see Table 1). The experimental design was conducted using a stratified holdout procedure with 30 runs, where approximately 75% of the patterns were randomly selected for the training set and the remaining 25% for the generalization set. All the data have been standardized and the experiments have been conducted using Matlab R2009a running on a Ubuntu Server (x86\_64 architecture) on a Intel Xeon at 2.00GHz with 8 Gb RAM.

### 3.1 Machine learning methods used for comparison purposes

The experimental section compares two basically different methodologies with different extensions for training MLP neural networks. The first group of classifiers are variations of the Evolutionary ELM (results are also compared to the original ELM and OPELM [19]):

- EELM. This method is set up with two fitness functions:  $CCR$  (EELM(C)) and  $MS$  (EELM(S)).
- EELMCS. This algorithm is set up with the three fitness functions proposed in Section 2.2:  $F_{\lambda CS}$  (EELMCS(CS)),  $F_{\lambda R}$  (EELMCS(R)) and  $F_{\lambda E}$  (EELMCS(E)).

The second type of neural network training algorithm is the Memetic Pareto Differential Evolution Neural Network (MPDENN) presented in [11]. MPDENN is a Multi-Objective Evolutionary Algorithm (MOEA) based on the Pareto Differential Evolution algorithm (PDE) presented by [2, 1]. The MPDENN trains ANNs considering  $C$  and  $MS$  as conflicting objectives which should be simultaneously optimized. In addition, the MPDENN applies a local search procedure to some individuals in the population. The local search algorithm used is the improved Resilient Backpropagation (iRprop<sup>+</sup>) algorithm [16]. Moreover, local search can improve classification performance although it is computationally costly. MPDENN can be used without local search; when doing so, we will refer to it as PDENN.

In addition, the experiments include two additional popular learning algorithms: a standard MLP trained with Resilient backpropagation (Rprop) algorithm [21] and the Support Vector Machine (SVM) [10, 25]. The Rprop Matlab's implementation is used for the former algorithm, while Cost Support Vector Clasificación (SVC) available in libSVM 3.0 [7] is used as the SVM classifier implementation.

All the ELM-based methods were trained with different algorithms (ELM, OPELM, EELM and EELMCS) using the Sigmoid function as the basis function. For ELM and OPELM, the number of hidden nodes gradually increases in intervals of 5 within the interval [5, 200] and the nearly optimal number of nodes for ELM and OPELM are then selected based on the 10-fold cross-validation method using the training set. For the E-ELM-CS(R), the range of the interval has been reduced for cross-validation of the number of hidden nodes to [5, 20]. PDENN and MPDENN automatically prune nodes, so a range of maximum and minimum numbers of hidden nodes must be provided. We have used the range [1, 6] according to the author's recommendations. Regarding Rprop, a nested cross-validation was also performed using a range of hidden nodes of [1, 30] with an interval of

one. The radial basis kernel was used with the SVC method. For the selection of SVC hyperparameters (regularization parameter,  $C$ , and width of the Gaussian functions,  $\gamma$ ), a grid search was performed with a 10-fold cross-validation, using the following ranges:  $C \in \{10^2, 10^1, \dots, 10^{-1}\}$  and  $\gamma \in \{10^2, 10^0, \dots, 10^{-8}\}$ . For all the methods, except PDENN and MPDENN, the optimal hyperparameters  $\hat{\theta}$  cross-validation criteria was the following:

$$\hat{\theta} = \arg \max_{\theta_i} \frac{\overline{C_{\theta_i}} + \overline{MS_{\theta_i}}}{2}. \quad (13)$$

### 3.2 Statistical results

Tables 4 and 5 in Appendix A present results in values of the mean and the standard deviation (SD) for  $\%C_G$ ,  $\%MS_G$  and  $T$  (in seconds) for 30 runs. Subindex G in  $\%C_G$  and  $\%MS_G$  indicates that results belongs to the generalization dataset. For these tables and Table 2 the best result is in bold face and the second best result is in italics.

Table 2: Mean statistical results and average rankings

Method	$\overline{C_G}(\%)$	$\overline{R_{C_G}}$	$\overline{MS_G}(\%)$	$\overline{R_{MS_G}}$	$\overline{T(secs.)}$	$\overline{R_T}$
EELMCS(R)	86.19	5.50	<b>58.86</b>	<b>2.45</b>	1.42E+002	7.00
EELMCS(E)	<i>86.87</i>	5.50	56.52	4.95	1.46E+002	8.20
EELMCS(CS)	86.47	6.50	<i>58.50</i>	<i>4.75</i>	1.39E+002	7.00
EELM(C)	86.78	<i>5.40</i>	48.81	6.75	1.41E+002	7.00
EELM(MS)	84.86	8.60	57.54	5.20	1.40E+002	6.80
OPELM	85.71	7.10	42.70	10.35	3.32E+000	3.70
ELM	86.09	7.20	44.33	7.90	<b>9.12E-002</b>	<i>1.70</i>
PDE(C)	82.92	10.25	41.81	9.45	2.08E+003	10.05
PDE(MS)	82.06	10.55	51.62	7.60	2.08E+003	10.35
HPDE(C)	86.07	6.90	46.91	7.25	1.37E+006	12.25
HPDE(MS)	84.89	8.30	55.60	6.35	1.37E+006	12.35
Rprop	85.06	7.20	38.05	11.20	8.39E-001	3.30
SVC	<b>88.60</b>	<b>2.00</b>	53.97	6.80	<i>1.44E-001</i>	<b>1.30</b>

The mean rankings of  $C_G$ ,  $MS_G$  and  $T$  are obtained to compare the different methods (see Table 2). A Friedman's non-parametric test for a significance level of  $\alpha = 0.1$  has been carried out to determine the statistical significance of the differences in rank in each method. The test rejected the null-hypothesis stating that all algorithms performed equally in the mean ranking of  $C_G$ ,  $MS_G$  and  $T$  so a Nemenyi post-hoc test [12] ( $\alpha = 0.1$ ) was used to compare all the methods and their variations. Figure 2 shows Critical Difference (CD) diagrams proposed in [12]. Subfigure 2a shows that there are two groups of classifiers regarding  $C_G$ . SVC has the best  $C_G$  ranking mean but it does not have significant differences compared with EELM variants but with EELM(MS). Regarding,  $MS_G$ , all the methods but Rprop, OPELM, PDE(C) and ELM have no significant differences when they are compared to one another. Regarding  $T$ , all the non-evolutionary approaches have similar performance, in addition, EELM methods (except EELMCS(E)) show

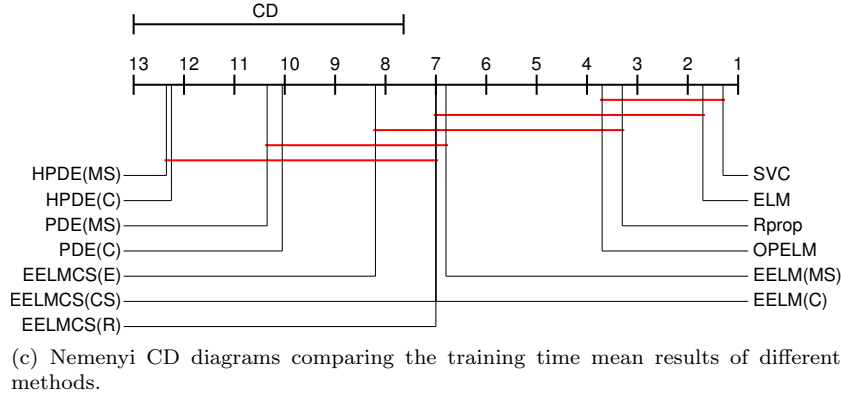
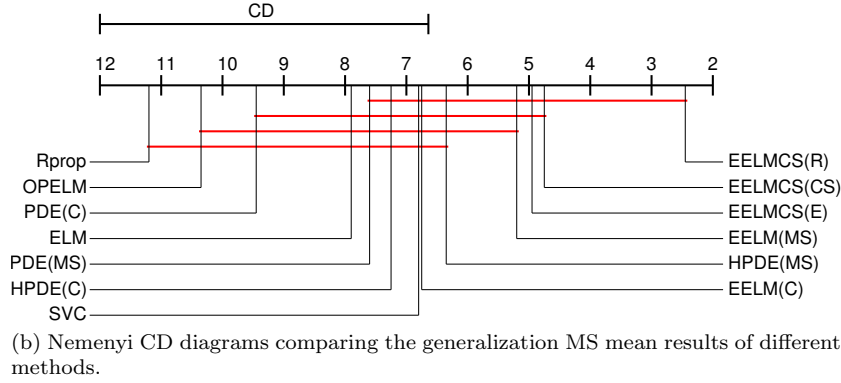
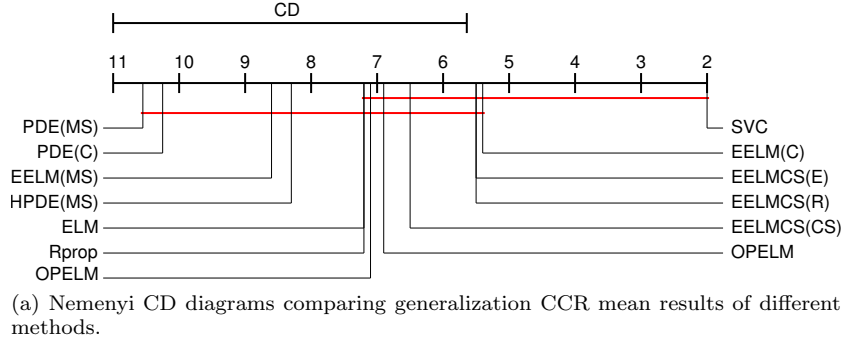


Fig. 2: Ranking tests for CCR, MS and training time.

similar computational time. As expected, the Pareto based solutions have the highest computational cost (especially the hybrid approaches).

Based on the robustness regarding  $C_G$  and  $MS_G$ , and considering computational cost, EELMCS(R) promises to be the best alternative. However, when comparing them to each other for  $MS_G$ , EELMCS(R) shows no significant differences with EELM(C), so our proposal cannot be justified. From a statistical point of view, this can be justified considering the effects of the results of com-

i	Algorithm	z	p	$\alpha'_{Holm}$
1	Rprop	5.02398	0.00000	0.00833
2	OPELM	4.53594	0.00001	0.00909
3	PDE(C)	4.01918	0.00006	0.01000
4	ELM	3.12922	0.00175	0.01111
5	PDE(MS)	2.95697	0.00311	0.01250
6	HPDE(C)	2.75601	0.00585	0.01429
7	SVC	2.49764	0.01250	0.01667
8	EELM(C)	2.46893	0.01355	0.02000
9	HPDE(MS)	2.23926	0.02514	0.02500
10	EELM(MS)	1.57897	0.11434	0.03333
11	EELMCS(E)	1.43542	0.15117	0.05000
12	EELMCS(CS)	1.32059	0.18664	0.10000

Table 3: Table with the different algorithms compared with the EELMCS(R) (i.e. the Control Algorithm) using the Holm procedure in terms of  $MS_G$ .

petitive methods, that is, the presence of EELM related methods. Therefore, the more powerful Holm post-hoc test is used to compare EELMCS(R) to all the other classifiers in order to justify our proposal. The Holm test is a multiple comparison procedure that can work with a control algorithm and compares it to the remaining methods [12]. The test statistics for comparing the  $i$ -th and the  $j$ -th method using this procedure is

$$z = \frac{R_i - R_j}{\sqrt{\frac{k(k+1)}{6N}}} \quad (14)$$

where  $k$  is the number of algorithms and  $N$  the number of datasets. The  $z$  value is used to find the corresponding probability from the table of normal distribution, which is then compared to an appropriate level of confidence  $\alpha$ . Holm's test adjusts the value for  $\alpha$  in order to compensate for multiple comparisons.

The results of the Holm tests ( $\alpha = 0.1$ ) for  $MS_G$  can be seen in Table 3, using the corresponding  $p$  and adjusted  $\alpha$  ( $\alpha'_{Holm}$ ) values. The EELMCS(R) is used as the Control Method. The horizontal line shows the division between methods significantly different from EELMCS(R) (in terms of  $MS_G$  when  $p < \alpha'_{Holm}$ ) and methods which are not significantly different. Considering the results of these tests, it can be concluded that the EELMCS(R) algorithm obtains a significantly higher ranking of  $MS_G$  when compared to most of the remaining methods, especially ELM, OPELM and EELM(C). Standard methods such as Rprop or SVC are not competitive when considering  $MS$ .

#### 4 Conclusions

This paper presents an efficient alternative to the Pareto based approach to train multi-class classifiers with a simultaneous improvement in  $C$  and  $MS$ .

Three different fitness functions were evaluated by extending the Evolutionary Extreme Learning Machine algorithm for training ANNs and were compared with different machine learning methodologies. Continuous fitness functions have proved to be more robust and suitable for evolutionary algorithms (see results details in Tables 4 and 5).

Statistical tests demonstrate that experimentally the E-ELM-CS(R) methodology gets similar results in *CCR* with respect to the other methods. However, statistical tests for *MS* demonstrate experimentally that it is significantly different than most algorithms. Considering the methods with the best significant results in *CCR* and *MS*, and considering the statistical test for training time *T*, we conclude that the weighted combination of global RMSE and the worst classified class RMSE give competitive results.

*MS* results in anneal, balance, glass or hepatitis datasets (see Tables 4 and 5) show that our proposal is specially suitable for problems with high number of classes and/or with small size classes, i.e. imbalanced datasets. Experimental results show that some methods focus only on the bigger classes.

**Acknowledgements** This work has been partially subsidized by the TIN 2008-06681-C06-03 project of the Spanish Inter-Ministerial Commission of Science and Technology (MICYT), FEDER funds and the P08-TIC-3745 project of the “Junta de Andalucía” (Spain). The research of Javier Sánchez-Monedero has been funded by the “Junta de Andalucía” Ph. D. Student Program.

## References

1. Abbass, H.A.: A memetic pareto evolutionary approach to artificial neural networks. In: M. Brooks, D. Corbet, M. Stumptner (eds.) AI2001, pp. 1–12. LNAI 2256, Springer-Verlag (2001)
2. Abbass, H.A., Sarker, R., Newton, C.: PDE: a Pareto-frontier differential evolution approach formulti-objective optimization problems. In: Proceedings of the 2001 Congress on Evolutionary Computation, vol. 2. Seoul, South Korea (2001)
3. Asuncion, A., Newman, D.: UCI machine learning repository (2007). URL <http://www.ics.uci.edu/~mllearn/MLRepository.html>
4. Bishop, C.M.: Neural Networks for Pattern Recognition, 1 edn. Oxford University Press, USA (1996)
5. Bishop, C.M.: Pattern Recognition and Machine Learning, 1st ed. 2006. corr. 2nd printing edn. Springer (2007)
6. Bridle, J.: Probabilistic interpretation of feedforward classification network outputs, with relationships to statistical pattern recognition. Neurocomputing: Algorithms, Architectures and Applications pp. 227–236 (1990)
7. Chang, C.C., Lin, C.J.: LIBSVM: a library for support vector machines (2001). Software available at <http://www.csie.ntu.edu.tw/~cjlin/libsvm>
8. Chen, L., Zhou, L., Pung, H.: Universal approximation and qos violation application of extreme learning machine. Neural Processing Letters **28**, 81–95 (2008). URL <http://dx.doi.org/10.1007/s11063-008-9083-z>. 10.1007/s11063-008-9083-z
9. Coello, C.A.: An updated survey of ga-based multiobjective optimization techniques. ACM Computer Surveys **32**(2), 109–143 (2000)
10. Cortes, C., Vapnik, V.: Support-vector networks. Machine Learning **20**(3), 273–297 (1995). Cited By (since 1996) 3823
11. Cruz-Ramírez, M., Sánchez-Monedero, J., Fernández-Navarro, F., Fernández, J., Hervás-Martínez, C.: Memetic pareto differential evolutionary artificial neural networks to determine growth multi-classes in predictive microbiology. Evolutionary Intelligence pp. 1–13 (2010)
12. Demšar, J.: Statistical comparisons of classifiers over multiple data sets. J. Mach. Learn. Res. **7**, 1–30 (2006)
13. Fernández-Caballero, J.C., Martínez-Estudillo, F.J., Hervás-Martínez, C., Gutiérrez, P.A.: Sensitivity versus accuracy in multiclass problems using memetic pareto evolutionary neural networks. IEEE Transactions on Neural Networks **21**(5), 750–770 (2010)
14. Huang, G.B., Chen, L., Siew, C.K.: Universal approximation using incremental constructive feedforward networks with random hidden nodes. IEEE Transactions on Neural Networks **17**(4), 879–892 (2006)

15. Huang, G.B., Zhu, Q.Y., Siew, C.K.: Extreme learning machine: Theory and applications. *Neurocomputing* **70**(1-3), 489 – 501 (2006)
16. Igel, C., Hsken, M.: Empirical evaluation of the improved rprop learning algorithms. *Neurocomputing* **50**(6), 105–123 (2003)
17. Ilonen, J., Kamarainen, J.K., Lampinen, J.: Differential evolution training algorithm for feed-forward neural networks. *Neural Processing Letters* **17**(1), 93–105 (2003)
18. Martínez-Estudillo, F.J., Gutiérrez, P.A., Hervás-Martínez, C., Fernández, J.C.: Evolutionary learning by a sensitivity-accuracy approach for multi-class problems. In: *Proceedings of the 2008 IEEE Congress on Evolutionary Computation (CEC'08)*, pp. 1581–1588. IEEE Press, Hong Kong, China (2008)
19. Miche, Y., Sorjamaa, A., Bas, P., Simula, O., Jutten, C., Lendasse, A.: OP-ELM: Optimally Pruned Extreme Learning Machine. *IEEE Transactions on Neural Networks* **21**(1), 158–162 (2010)
20. Provost, F., Fawcett, T.: Analysis and visualization of classifier performance: Comparison under imprecise class and cost distributions. In: *In Proc. of the 3rd International Conference on Knowledge Discovery and Data Mining*, pp. 43–48 (1997)
21. Riedmiller, M., Braun, H.: A direct adaptive method for faster backpropagation learning: The RPROP algorithm. In: *Proceedings of the 1993 IEEE International Conference on Neural Networks*, pp. 586–591. San Francisco, CA (1993)
22. Roth, S., Gepperth, A., Igel, C.: Multi-objective neural network optimization for visual object detection. In: Y. Jin (ed.) *Multi-Objective Machine Learning, Studies in Computational Intelligence*, vol. 16, pp. 629–655. Springer Berlin / Heidelberg (2006)
23. Sánchez-Monedero, J., Hervás-Martínez, C., Martínez-Estudillo, F., Ruz, M., Moreno, M., Cruz-Ramírez, M.: Evolutionary Learning Using a Sensitivity-Accuracy Approach for Classification. In: *Hybrid Artificial Intelligence Systems, Lecture Notes in Computer Science*, vol. 6077, pp. 288–295. Springer Berlin / Heidelberg (2010)
24. Storn, R., Price, K.: Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization* **11**(4), 341–359 (1997)
25. Vapnik, V.: An overview of statistical learning theory. *IEEE Transactions on Neural Networks* **10**(5), 988–999 (1999)
26. Zhang, R., Huang, G.B., Sundararajan, N., Saratchandran, P.: Multi-category classification using an extreme learning machine for microarray gene expression cancer diagnosis. *IEEE/ACM Transactions on Computational Biology and Bioinformatics* **4**(3), 485–495 (2007)
27. Zhu, Q.Y., Qin, A., Suganthan, P., Huang, G.B.: Evolutionary extreme learning machine. *Pattern Recognition* **38**(10), 1759 – 1763 (2005)

## A Statistical results tables for $C$ , $MS$ and $T$

Table 4: Statistical results for  $C$ ,  $MS$  and  $T$ 

Dataset	Algorithm	$C$	$MS$	$T$
anneal	EELMCS(R)	96.46 $\pm$ 01.75	80.43 $\pm$ 11.13	1.71E+002 $\pm$ 1.84E+000
	EELMCS(E)	95.48 $\pm$ 01.93	76.76 $\pm$ 11.14	1.79E+002 $\pm$ 3.99E+000
	EELMCS(CS)	98.07 $\pm$ 01.71	<b>89.01 <math>\pm</math> 11.49</b>	1.62E+002 $\pm$ 2.59E+000
	EELM(C)	<b>99.11 <math>\pm</math> 00.90</b>	59.95 $\pm$ 47.29	1.67E+002 $\pm$ 7.36E+000
	EELM(MS)	96.07 $\pm$ 02.65	83.61 $\pm$ 13.90	1.61E+002 $\pm$ 2.47E+000
	OPELM	95.87 $\pm$ 01.21	51.67 $\pm$ 06.48	5.97E+000 $\pm$ 8.86E-001
	ELM	96.74 $\pm$ 01.01	55.60 $\pm$ 12.83	2.26E-001 $\pm$ 9.96E-002
	PDE(C)	90.24 $\pm$ 03.23	34.40 $\pm$ 27.22	3.02E+003 $\pm$ 3.92E+002
	PDE(MS)	86.22 $\pm$ 06.42	53.76 $\pm$ 15.37	3.02E+003 $\pm$ 3.92E+002
	HPDE(C)	92.22 $\pm$ 02.56	41.40 $\pm$ 27.91	3.01E+003 $\pm$ 4.77E+002
	HPDE(MS)	87.08 $\pm$ 06.81	59.14 $\pm$ 14.01	3.01E+003 $\pm$ 4.77E+002
	Rprop	95.82 $\pm$ 00.03	19.47 $\pm$ 00.37	2.24E+000 $\pm$ 8.14E-001
	SVC	97.78 $\pm$ 00.00	50.00 $\pm$ 00.00	<b>4.84E-002 <math>\pm</math> 0.00E+000</b>
balance	EELMCS(R)	91.65 $\pm$ 00.94	<b>87.67 <math>\pm</math> 06.83</b>	6.14E+001 $\pm$ 2.41E+000
	EELMCS(E)	91.86 $\pm$ 00.79	87.42 $\pm$ 06.78	6.38E+001 $\pm$ 2.08E+000
	EELMCS(CS)	90.92 $\pm$ 01.47	83.32 $\pm$ 10.85	6.07E+001 $\pm$ 1.29E+000
	EELM(C)	91.32 $\pm$ 01.70	36.33 $\pm$ 26.46	6.10E+001 $\pm$ 1.67E+000
	EELM(MS)	90.49 $\pm$ 02.00	81.86 $\pm$ 19.57	5.91E+001 $\pm$ 3.10E+000
	OPELM	91.97 $\pm$ 01.61	16.33 $\pm$ 18.29	6.89E+000 $\pm$ 1.17E+000
	ELM	88.55 $\pm$ 01.39	06.67 $\pm$ 06.06	3.54E-001 $\pm$ 1.07E-001
	PDE(C)	90.36 $\pm$ 01.30	23.33 $\pm$ 16.68	1.03E+002 $\pm$ 1.03E+001
	PDE(MS)	91.05 $\pm$ 01.15	85.15 $\pm$ 07.68	1.03E+002 $\pm$ 1.03E+001
	HPDE(C)	91.24 $\pm$ 01.23	29.00 $\pm$ 11.85	1.20E+002 $\pm$ 1.34E+001
	HPDE(MS)	91.22 $\pm$ 01.49	84.62 $\pm$ 06.82	1.20E+002 $\pm$ 1.34E+001
	Rprop	92.56 $\pm$ 00.04	17.33 $\pm$ 00.27	8.41E-001 $\pm$ 2.89E-001
	SVC	<b>93.59 <math>\pm</math> 00.00</b>	80.00 $\pm$ 00.00	<b>1.19E-002 <math>\pm</math> 0.00E+000</b>
breastw	EELMCS(R)	96.30 $\pm$ 00.93	<b>94.46 <math>\pm</math> 02.07</b>	1.77E+001 $\pm$ 3.43E-001
	EELMCS(E)	96.02 $\pm$ 00.79	93.24 $\pm$ 02.11	1.88E+001 $\pm$ 4.19E-001
	EELMCS(CS)	96.23 $\pm$ 00.86	93.54 $\pm$ 02.23	1.79E+001 $\pm$ 3.90E-001
	EELM(C)	96.38 $\pm$ 00.77	94.42 $\pm$ 01.79	1.72E+001 $\pm$ 4.41E-001
	EELM(MS)	95.70 $\pm$ 00.92	91.69 $\pm$ 02.52	1.73E+001 $\pm$ 3.60E-001
	OPELM	95.92 $\pm$ 00.88	93.76 $\pm$ 01.98	1.40E+000 $\pm$ 2.97E-001
	ELM	95.81 $\pm$ 00.66	92.06 $\pm$ 01.89	1.96E-002 $\pm$ 2.53E-002
	PDE(C)	95.14 $\pm$ 00.99	90.22 $\pm$ 02.69	2.47E+001 $\pm$ 3.56E+000
	PDE(MS)	95.18 $\pm$ 00.93	90.33 $\pm$ 02.85	2.47E+001 $\pm$ 3.56E+000
	HPDE(C)	95.03 $\pm$ 00.86	89.56 $\pm$ 02.19	2.92E+001 $\pm$ 3.77E+000
	HPDE(MS)	94.93 $\pm$ 00.85	89.44 $\pm$ 02.16	2.92E+001 $\pm$ 3.77E+000
	Rprop	96.13 $\pm$ 00.01	93.43 $\pm$ 00.02	4.78E-001 $\pm$ 1.46E-001
	SVC	<b>96.57 <math>\pm</math> 00.00</b>	91.67 $\pm$ 00.00	<b>7.26E-003 <math>\pm</math> 0.00E+000</b>
card	EELMCS(R)	88.07 $\pm$ 01.64	<b>86.09 <math>\pm</math> 01.59</b>	4.95E+001 $\pm$ 3.95E-001
	EELMCS(E)	88.13 $\pm$ 01.55	85.95 $\pm$ 03.12	5.22E+001 $\pm$ 3.43E-001
	EELMCS(CS)	87.15 $\pm$ 01.94	85.59 $\pm$ 02.25	4.85E+001 $\pm$ 1.59E+000
	EELM(C)	86.92 $\pm$ 01.50	84.82 $\pm$ 02.50	4.74E+001 $\pm$ 8.91E-001
	EELM(MS)	87.23 $\pm$ 01.62	85.02 $\pm$ 02.28	4.86E+001 $\pm$ 1.39E+000
	OPELM	84.84 $\pm$ 01.84	82.93 $\pm$ 02.72	6.21E+000 $\pm$ 7.78E-001
	ELM	85.53 $\pm$ 01.93	84.33 $\pm$ 02.03	5.02E-002 $\pm$ 5.34E-002
	PDE(C)	85.39 $\pm$ 02.32	83.01 $\pm$ 03.08	2.98E+001 $\pm$ 7.36E+000
	PDE(MS)	85.74 $\pm$ 02.06	84.08 $\pm$ 02.44	2.98E+001 $\pm$ 7.36E+000
	HPDE(C)	86.55 $\pm$ 01.14	84.85 $\pm$ 02.25	5.37E+001 $\pm$ 1.10E+001
	HPDE(MS)	86.51 $\pm$ 01.32	85.30 $\pm$ 01.83	5.37E+001 $\pm$ 1.10E+001
	Rprop	86.19 $\pm$ 00.08	82.85 $\pm$ 00.16	6.06E-001 $\pm$ 1.49E-001
	SVC	<b>88.44 <math>\pm</math> 00.00</b>	85.42 $\pm$ 00.00	<b>3.31E-002 <math>\pm</math> 0.00E+000</b>
gene	EELMCS(R)	84.73 $\pm$ 01.15	78.96 $\pm$ 02.14	9.79E+002 $\pm$ 8.23E+001
	EELMCS(E)	84.67 $\pm$ 01.16	78.20 $\pm$ 03.59	9.99E+002 $\pm$ 8.63E+001
	EELMCS(CS)	83.46 $\pm$ 01.30	78.84 $\pm$ 03.80	9.42E+002 $\pm$ 8.57E+001
	EELM(C)	84.50 $\pm$ 01.45	78.52 $\pm$ 03.34	9.61E+002 $\pm$ 8.95E+001
	EELM(MS)	83.69 $\pm$ 01.65	79.57 $\pm$ 02.60	9.61E+002 $\pm$ 9.05E+001
	OPELM	77.99 $\pm$ 01.41	62.86 $\pm$ 04.43	8.56E+000 $\pm$ 1.06E-001
	ELM	80.50 $\pm$ 01.33	69.65 $\pm$ 03.23	<b>2.37E-001 <math>\pm</math> 2.09E-002</b>
	PDE(C)	70.37 $\pm$ 04.18	56.74 $\pm$ 09.93	1.71E+004 $\pm$ 4.38E+003
	PDE(MS)	69.99 $\pm$ 04.34	65.25 $\pm$ 05.83	1.71E+004 $\pm$ 4.38E+003
	HPDE(C)	82.32 $\pm$ 02.83	75.15 $\pm$ 05.43	1.37E+007 $\pm$ 2.70E+006
	HPDE(MS)	82.06 $\pm$ 02.76	74.89 $\pm$ 05.47	1.37E+007 $\pm$ 2.70E+006
	Rprop	84.39 $\pm$ 00.07	73.52 $\pm$ 00.23	2.17E+000 $\pm$ 5.43E-001
	SVC	<b>90.92 <math>\pm</math> 00.00</b>	<b>89.32 <math>\pm</math> 00.00</b>	1.33E+000 $\pm$ 0.00E+000

Table 5: Statistical results for  $C$ ,  $MS$  and  $T$ 

Dataset	Algorithm	$C$	$MS$	$T$
glass	EELMCS(R)	62.83 $\pm$ 07.82	<b>19.17 <math>\pm</math> 16.54</b>	7.31E+001 $\pm$ 9.53E-001
	EELMCS(E)	69.62 $\pm$ 04.16	05.00 $\pm$ 12.11	7.52E+001 $\pm$ 1.05E+000
	EELMCS(CS)	68.55 $\pm$ 04.41	18.61 $\pm$ 17.46	7.87E+001 $\pm$ 1.37E+000
	EELM(C)	69.69 $\pm$ 05.22	07.78 $\pm$ 13.83	7.81E+001 $\pm$ 1.64E+000
	EELM(MS)	66.42 $\pm$ 06.08	15.47 $\pm$ 17.16	7.74E+001 $\pm$ 1.76E+000
	OPELM	<b>71.70 <math>\pm</math> 03.43</b>	02.50 $\pm$ 07.63	2.88E+000 $\pm$ 4.94E-001
	ELM	70.44 $\pm$ 04.86	00.00 $\pm$ 00.00	<b>4.38E-003 <math>\pm</math> 1.42E-002</b>
	PDE(C)	61.89 $\pm$ 08.53	01.67 $\pm$ 06.34	3.26E+002 $\pm$ 5.04E+001
	PDE(MS)	57.99 $\pm$ 09.11	09.46 $\pm$ 15.06	3.26E+002 $\pm$ 5.04E+001
	HPDE(C)	69.18 $\pm$ 05.23	02.50 $\pm$ 07.63	3.61E+002 $\pm$ 7.80E+001
	HPDE(MS)	64.53 $\pm$ 07.41	18.43 $\pm$ 17.96	3.61E+002 $\pm$ 7.80E+001
	Rprop	59.69 $\pm$ 00.11	00.00 $\pm$ 00.00	5.52E-001 $\pm$ 1.80E-001
	SVC	64.15 $\pm$ 00.00	00.00 $\pm$ 00.00	8.35E-003 $\pm$ 0.00E+000
hepatitis	EELMCS(R)	74.27 $\pm$ 04.23	42.92 $\pm$ 13.80	1.71E+000 $\pm$ 8.55E-002
	EELMCS(E)	76.24 $\pm$ 04.26	38.75 $\pm$ 14.44	1.70E+000 $\pm$ 9.52E-003
	EELMCS(CS)	77.18 $\pm$ 04.43	45.90 $\pm$ 12.95	1.67E+000 $\pm$ 1.06E-002
	EELM(C)	76.41 $\pm$ 04.64	35.00 $\pm$ 15.54	1.67E+000 $\pm$ 8.34E-003
	EELM(MS)	74.87 $\pm$ 04.92	41.25 $\pm$ 13.99	1.67E+000 $\pm$ 9.07E-003
	OPELM	76.32 $\pm$ 03.79	30.00 $\pm$ 14.53	2.27E-001 $\pm$ 6.25E-003
	ELM	76.75 $\pm$ 03.92	32.50 $\pm$ 11.65	<b>1.53E-003 <math>\pm</math> 3.65E-005</b>
	PDE(C)	74.70 $\pm$ 04.24	33.33 $\pm$ 13.67	1.14E+001 $\pm$ 1.78E+000
	PDE(MS)	74.70 $\pm$ 04.35	35.42 $\pm$ 12.32	1.14E+001 $\pm$ 1.78E+000
	HPDE(C)	75.38 $\pm$ 03.34	31.67 $\pm$ 12.17	1.40E+001 $\pm$ 3.38E+000
	HPDE(MS)	75.21 $\pm$ 04.11	33.33 $\pm$ 12.43	1.40E+001 $\pm$ 3.38E+000
	Rprop	76.07 $\pm$ 00.04	22.08 $\pm$ 00.20	2.59E-001 $\pm$ 8.00E-002
	SVC	<b>79.49 <math>\pm</math> 00.00</b>	<b>50.00 <math>\pm</math> 00.00</b>	1.58E-003 $\pm$ 0.00E+000
iris	EELMCS(R)	96.00 $\pm$ 01.48	89.16 $\pm$ 03.29	1.76E+000 $\pm$ 2.17E-002
	EELMCS(E)	95.70 $\pm$ 01.64	88.65 $\pm$ 03.08	1.82E+000 $\pm$ 1.78E-002
	EELMCS(CS)	94.96 $\pm$ 01.93	87.56 $\pm$ 03.81	1.80E+000 $\pm$ 8.17E-003
	EELM(C)	95.26 $\pm$ 02.16	89.09 $\pm$ 04.04	1.81E+000 $\pm$ 1.10E-002
	EELM(MS)	94.74 $\pm$ 01.89	87.32 $\pm$ 03.63	1.80E+000 $\pm$ 1.07E-002
	OPELM	92.89 $\pm$ 03.16	86.98 $\pm$ 07.09	8.08E-001 $\pm$ 2.09E-001
	ELM	95.70 $\pm$ 02.18	90.54 $\pm$ 05.07	1.29E-002 $\pm$ 1.55E-002
	PDE(C)	96.58 $\pm$ 01.82	90.26 $\pm$ 04.01	7.98E+000 $\pm$ 9.75E-001
	PDE(MS)	95.81 $\pm$ 01.43	87.69 $\pm$ 04.33	7.98E+000 $\pm$ 9.75E-001
	HPDE(C)	97.09 $\pm$ 00.89	91.28 $\pm$ 02.66	9.89E+000 $\pm$ 2.43E+000
	HPDE(MS)	95.73 $\pm$ 01.23	87.18 $\pm$ 03.69	9.89E+000 $\pm$ 2.43E+000
	Rprop	90.00 $\pm$ 00.13	71.76 $\pm$ 00.36	4.20E-001 $\pm$ 1.21E-001
	SVC	<b>97.78 <math>\pm</math> 00.00</b>	<b>93.33 <math>\pm</math> 00.00</b>	<b>6.18E-004 <math>\pm</math> 0.00E+000</b>
lymph	EELMCS(R)	79.82 $\pm$ 03.74	04.83 $\pm$ 18.41	2.95E+001 $\pm$ 2.00E-001
	EELMCS(E)	78.02 $\pm$ 05.35	02.33 $\pm$ 12.78	3.10E+001 $\pm$ 2.06E-001
	EELMCS(CS)	77.75 $\pm$ 05.10	02.67 $\pm$ 14.61	3.11E+001 $\pm$ 3.93E-001
	EELM(C)	79.01 $\pm$ 05.10	00.00 $\pm$ 00.00	3.13E+001 $\pm$ 3.64E-001
	EELM(MS)	70.45 $\pm$ 06.46	06.28 $\pm$ 19.29	3.12E+001 $\pm$ 2.55E-001
	OPELM	82.43 $\pm$ 04.36	00.00 $\pm$ 00.00	1.79E-001 $\pm$ 4.19E-003
	ELM	79.28 $\pm$ 04.83	07.00 $\pm$ 21.36	4.85E-003 $\pm$ 1.22E-004
	PDE(C)	79.46 $\pm$ 04.63	02.33 $\pm$ 12.78	9.75E+001 $\pm$ 2.67E+001
	PDE(MS)	79.28 $\pm$ 04.78	02.33 $\pm$ 12.78	9.75E+001 $\pm$ 2.67E+001
	HPDE(C)	80.99 $\pm$ 05.74	<b>14.83 <math>\pm</math> 30.44</b>	1.21E+002 $\pm$ 2.57E+001
	HPDE(MS)	80.81 $\pm$ 05.74	<b>14.83 <math>\pm</math> 30.44</b>	1.21E+002 $\pm$ 2.57E+001
	Rprop	80.99 $\pm$ 00.10	00.00 $\pm$ 00.00	3.21E-001 $\pm$ 1.29E-001
	SVC	<b>83.78 <math>\pm</math> 00.00</b>	00.00 $\pm$ 00.00	<b>2.84E-003 <math>\pm</math> 0.00E+000</b>
zoo	EELMCS(R)	91.73 $\pm$ 04.06	<b>08.89 <math>\pm</math> 18.69</b>	3.61E+001 $\pm$ 3.84E-001
	EELMCS(E)	92.93 $\pm$ 04.54	<b>08.89 <math>\pm</math> 27.59</b>	3.76E+001 $\pm$ 2.98E-001
	EELMCS(CS)	90.40 $\pm$ 04.15	00.00 $\pm$ 00.00	4.22E+001 $\pm$ 3.24E-001
	EELM(C)	89.20 $\pm$ 03.95	02.22 $\pm$ 12.17	4.22E+001 $\pm$ 3.83E-001
	EELM(MS)	88.93 $\pm$ 04.42	03.33 $\pm$ 18.26	4.23E+001 $\pm$ 4.78E-001
	OPELM	87.20 $\pm$ 05.79	00.00 $\pm$ 00.00	4.60E-002 $\pm$ 7.87E-004
	ELM	91.60 $\pm$ 04.74	05.00 $\pm$ 20.13	1.68E-003 $\pm$ 1.30E-004
	PDE(C)	85.07 $\pm$ 06.12	02.78 $\pm$ 10.80	6.15E+001 $\pm$ 1.68E+001
	PDE(MS)	84.67 $\pm$ 06.31	02.78 $\pm$ 10.80	6.15E+001 $\pm$ 1.68E+001
	HPDE(C)	90.67 $\pm$ 05.49	<b>08.89 <math>\pm</math> 27.59</b>	7.98E+001 $\pm$ 3.89E+001
	HPDE(MS)	90.80 $\pm$ 05.37	<b>08.89 <math>\pm</math> 27.59</b>	7.98E+001 $\pm$ 3.89E+001
	Rprop	86.93 $\pm$ 00.12	00.00 $\pm$ 00.00	4.98E-001 $\pm$ 2.04E-001
	SVC	<b>96.00 <math>\pm</math> 00.00</b>	00.00 $\pm$ 00.00	<b>1.50E-003 <math>\pm</math> 0.00E+000</b>